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Heat transfer by mixed convection in a vertical flow undergoing transition

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Abstract—Heat transfer, for aiding mixed convection from vertical, uniform flux surfaces and for small forced convection effects, is considered here. Simple relations have been proposed to correlate the new experimental data which were obtained in a flow undergoing transition from a laminar regime toward turbulence. Experiments were performed in air at pressures ranging from 4.4 to about 8 bar. The correlation based on experimental data for laminar flow for $Pr = 0.7$ has been extended to other Prandtl numbers through numerical integration of the transport equations. It is shown that, for both laminar and turbulent mixed convection, the Nusselt number may be successfully correlated, employing suitable combinations of the corresponding heat transfer correlations for forced and for natural convection. The parameter characterizing the mixed convection effect was found to be different in laminar and turbulent flow. However, in each of these regions, the relevant parameter is proportional to the ratio of the applicable characteristic forced and natural convection velocity scales.

1. INTRODUCTION

COMBINED forced and natural convection, or mixed convection, commonly arises in nature and in technological applications such as cooling of electronic components, loss of coolant accidents in nuclear reactors [1], etc. The mechanisms of mixed convection from a vertical, uniform flux surface have been considered theoretically by Wilks [2] and also by Carey and Gebhart [3]. In ref. [3] experimental results are also presented. Babezha *et al.* [4] report measurements of heat transfer rates for mixed convection from a vertical, uniform flux surface. The other experimental studies of mixed convection, by Kliegel [5], Oosthuizen and Bassey [6], Gryzagoridis [7] and Ramachandran *et al.* [8], were with an isothermal surface.

All of the above studies consider only the laminar flow regime. Hall and Price [9] measured the effect of superimposing a forced, upward flow on a turbulent, natural convection boundary layer, again with aiding effects. The experiment was in air at atmospheric pressure in a vertical wind tunnel. Various levels of free-stream velocity were used. At low velocities, the heat transfer coefficient at a given downstream location was less than that for pure natural convection; at higher velocities it increased. A minimum heat transfer coefficient occurred when the imposed free-stream velocity was same as the peak velocity measured with pure natural convection. All these results are for turbulent flow.

The present set of experiments in air were at pressures in the range of 4.4 to about 8 bar and for various heat flux levels in the range 14–1300 W m^{-2}. This range encompasses laminar, transition and turbulent flow regimes. Simple relations are given to correlate the data. In the laminar regime, the correlation is based on experimental data for air ($Pr = 0.7$); it was extended to other values of Prandtl number by numerical integration of the governing equations.

2. EXPERIMENTAL SETUP

The test section is a 100-dm^3 pressure vessel, 76 cm high and 33 cm in diameter. The test surface assembly consists of two 0.013-mm-thick foils of Inconel 600, separated by layers of fabric, impregnated with a thermosetting resin. To measure the surface temperature, seven copper–constantan thermocouples, 0.05 mm in diameter, were fused into the layers of the fabric. The assembly was 40 cm high, 15 cm wide and 0.018 cm thick, when bonded.

The electrical circuit to heat the surface consisted of a 0–40 V, 0–40 A regulated DC power supply, connected in series with the test surface and a 0.01 Ω precision resistor, to measure the current through the surface. The voltage drop across the test surface was independently measured by means of a digital voltmeter. These two measurements determine the total flux dissipated by the surface. From the measured surface temperature distribution, the net radiative flux correction was estimated. In our experiments, the maximum value of radiative flux correction amounted to no more that 15% of the total heat input.

The air velocity in the test section was measured with a hot wire anemometer, DISA 55P14, calibrated using the method reported by Mahajan and Gebhart [10]. The air temperatures were measured with a 0.05-mm copper–constantan thermocouple. Both of these boundary-layer probes could be positioned at any location normal to the surface by means of a

NOMENCLATURE

- c_p specific heat at constant pressure
- $f_F(Pr)$ $[1+(0.0205/Pr)^{2/3}]^{1/4}$
- $f_N(Pr)$ $[1+(0.437/Pr)^{9/16}]^{16/9}$
- G^* $5(Gr_x^*/5)^{1/5}$
- Gr_x^* $g\beta q''x^4/k\nu^2$
- Gr_x $g\beta x^3(t_0-t_\infty)/\nu^2$
- g acceleration due to gravity
- h heat transfer coefficient
- k thermal conductivity
- n a number
- Nu Nusselt number, hx/k
- Pr Prandtl number, $\mu c_p/k$
- q'' flux dissipated by the surface
- Ra_x^* Rayleigh number, Gr_x^*Pr
- Re_x Reynolds number, $U_\infty x/\nu$
- t temperature [°C]
- $\bar{U}$ time-averaged, local tangential velocity in the boundary region
- U_∞ free-stream velocity
- U_c a characteristic, laminar, natural convection velocity
- $U_{c,t}$ a characteristic, turbulent, natural convection velocity
- x coordinate parallel to the surface and opposite to the direction of gravity.

Greek symbols
- β coefficient of thermal expansion, $(-1/\rho)(\partial\rho/\partial t)_p$
- ε_m mixed convection parameter, laminar flow, $\equiv Re_x/(G^*/5)^2$
- $\varepsilon_{m,t}$ mixed convection parameter, turbulent flow, $\equiv Re_x/Gr_x^{*0.33}$
- μ dynamic viscosity
- ν kinematic viscosity, μ/ρ
- ρ density.

Subscripts
- F forced convection
- F, t turbulent, forced convection
- f properties based on film temperature
- MC mixed convection
- MC, t turbulent, mixed convection
- N natural convection
- N, t turbulent, natural convection
- p at constant pressure
- x characteristic length scale
- 0 quantity evaluated at the surface
- ∞ quantity evaluated at a large distance from the surface.

DISA 55H01 traversing mechanism. The voltage output from these probes and the surface thermocouples were processed through a data acquisition system consisting of a digital voltmeter, a scanner and a microcomputer. Experiments were performed at pressure levels of 4.4, 6.1, 6.8 and 7.9 bars. At each pressure level, several values of heat flux were used.

3. RESULTS AND DISCUSSION

This section presents the results from both our experiments and numerical calculations. The experimental results in the laminar regime are considered first. An appropriate correlation was determined for the measured heat transfer coefficients in air. Next, the results of numerical calculations are presented. These results allowed the extension of the above correlation to other Prandtl numbers. Finally, the data in the transition and turbulent regimes are considered. A suitable correlation was determined for heat transfer coefficients in the fully turbulent region, incorporating the data in ref. [9].

3.1. *Laminar regime—experimental results*

In developing the correlation for the local heat transfer coefficient, $h(x)$, along the surface, it is convenient to employ the procedure of Churchill and Usagi [11]. This involves expressing Nu_{MC}, the local Nusselt number for mixed convection, as

$$Nu_{MC}^n = Nu_N^n + Nu_F^n \tag{1}$$

where the subscripts N and F indicate local natural and forced convection values, respectively, and n is some number. For air $Pr = 0.7$ and for a uniform flux surface

$$Nu_N = 0.53\sqrt[5]{Gr_x^*}, \quad \text{from [12]} \tag{2}$$

$$Nu_F = 0.40\sqrt{Re_x^*}, \quad \text{from [11]}. \tag{3}$$

Based on the form of equation (1), Churchill [13] developed a correlation for predicting heat transfer coefficients in laminar, mixed convection from a vertical, uniform flux surface. For $Pr = 0.7$ that correlation is written as

$$\frac{Nu_{MC}}{Nu_{FC}} = \left[1+\left(\frac{Nu_{NC}}{Nu_{FC}}\right)^3\right]^{1/3} \tag{4}$$

with

$$Nu_{NC} = 0.039Gr_x^{1/4} = 0.039\left[\frac{g\beta x^3(t_0-t_\infty)}{\nu^2}\right]^{1/4}. \tag{5}$$

For a uniform flux surface the known quantity is Gr_x^*, not Gr_x. The two, however, may be related as follows. By definition

$$Nu_{MC} \equiv \frac{hx}{k} = \frac{q''x}{k(t_0 - t_\infty)} = \frac{Gr_x^*}{Gr_x}. \quad (6)$$

Using equation (6), equation (5) is rewritten as

$$Nu_{NC} = 0.039\left(\frac{Gr_x^*}{Nu_{MC}}\right)^{1/4}. \quad (7)$$

By substituting equation (7) into (4), it is clear that the determination of Nu_{MC} would involve solving an algebraic equation. For the correlation given in ref. [13] the value on n in equation (1) is 3. From equation (1), for $Pr = 0.7$, using $n = 3$ and the two-component Nusselt number correlations, equations (2) and (3), the correlation in ref. [13] is recast for a uniform flux surface as

$$\frac{Nu_{MC}}{\sqrt{Re_x}} = \left(\frac{0.2909}{\varepsilon_m^{3/2}} + 0.0604\right)^{1/3}. \quad (8)$$

Here, ε_m is a parameter that characterizes the relative importance of forced and natural convection effects. It is defined as

$$\varepsilon_m \equiv Re_x/(G^*/5)^2 \equiv Re_x/(Gr_x^*/5)^{0.4}. \quad (9)$$

A value of $\varepsilon_m \ll 1$ implies a dominance of natural convection and $\varepsilon_m \gg 1$ implies that forced-convection effects are dominant. Also $\varepsilon_m = U_\infty/U_c$, where $U_c = \nu G^{*2}/25x$ is a characteristic laminar, natural convection velocity. On the other hand, with $n = 4$, equation (1) may be recast, for $Pr = 0.7$, as

$$\frac{Nu_{MC}}{\sqrt{Re_x}} = \left[\frac{0.1928}{\varepsilon_m^2} + 0.026\right]^{1/4}. \quad (10)$$

In Fig. 1, the new data in the laminar regime have been plotted in the form of $Nu_{MC}/\sqrt{Re_x}$ vs ε_m. For these data, the sensors detected no fluctuations in either the flow velocity or temperature. Both equations (8) and (10) are seen to correlate the data equally well, the r.m.s. of the deviation being about 4% for each.

Two other correlations have been proposed by prior investigators for this same circumstance. That of Babezha *et al.* [4] is

$$\frac{Nu_{MC}}{\sqrt{Re_x}} = 0.37 + 0.325/\varepsilon_m^{5/6}. \quad (11)$$

It is said to be valid in the range $0.005 < Gr_x^*/Re_x^{2.5} < 50$. In that range equation (11) agrees with (10) to within 5%, and with equation (8) to within 2%. However, it cannot be used at the natural convection limit, i.e. as $Re_x \to 0$. It then tends to an infinite value of the Nusselt number. The correlation proposed recently by Shai and Barnea [14] takes the form

$$\frac{Nu_{MC}}{\sqrt{Re_x}} = \left\{(0.483Pr_f^{1/2})^3 + \left[0.616\left(\frac{Ra^{*2/5}}{Re_x}\right)^{1/2}\left(\frac{Pr_f}{0.8+Pr_f}\right)^{1/5}\right]^3\right\}^{1/3}. \quad (12)$$

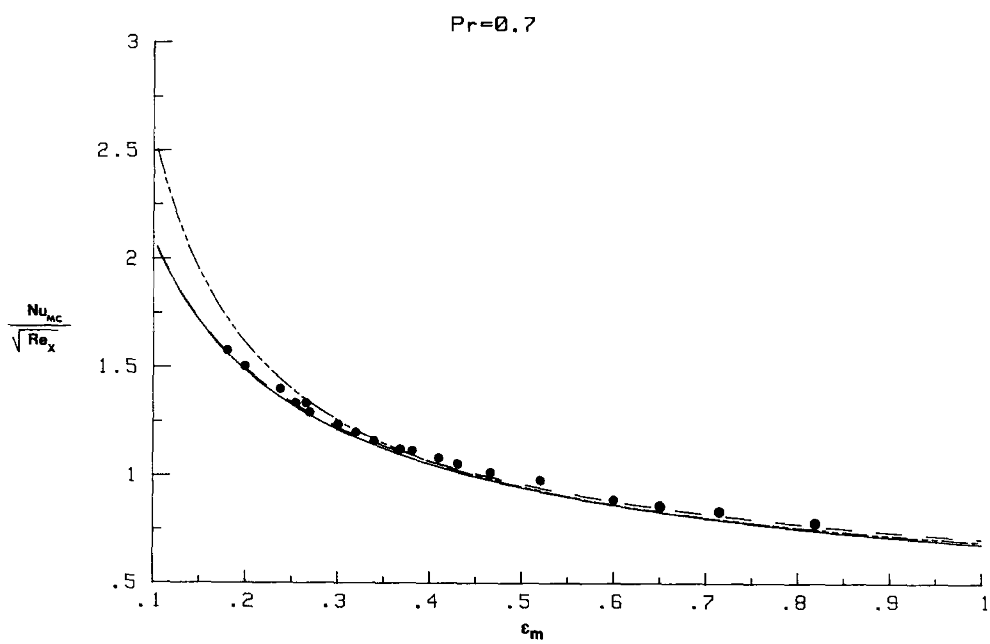

FIG. 1. Local heat transfer rates in laminar, mixed convection from a vertical, uniform flux surface, as a function of the mixed convection parameter, ε_M. ● experiment; —— equation (10); – – – – equation (8); — — — — equation (11).

In the range $0.4 \leqslant \varepsilon_m \leqslant 16$, the r.m.s difference between this prediction and those of equations (8), (10) and (11) at discrete values of ε_m, is 1%, 5% and 2%, respectively. This formula was derived from quite simple analysis and was expected to be valid for values of Pr around 1. However, as $Gr_x^* \to 0$ it yields

$$\frac{Nu}{\sqrt{Re_x}} \to 0.483 Pr^{1/2}.$$

In this limit, from ref. [15], the result from an integral method is

$$\frac{Nu}{\sqrt{Rx_x}} = 0.453 Pr^{1/3}.$$

Since equation (12) is also much more complicated, either equation (8) or equation (10) is recommended over the whole range, $0 \leqslant \varepsilon_m \leqslant \infty$, for $Pr = 0.07$.

3.2. *Laminar regime—numerical results*

The correlations given above apply for values of Pr around 0.7. It would certainly be useful to extend their applicability to other Prandtl number values. However, there have been no experiments for other Prandtl numbers. Wilks [2, 16] reported numerically calculated heat transfer rates over the range $0.01 \leqslant Pr \leqslant 100$. However, these results exhibit considerable scatter for Prandtl numbers differing from unity, see ref. [13].

The numerical results reported here were obtained by solving the governing equations through the box scheme developed by Keller [17]. This is an implicit scheme, details of which are found in ref. [18]. In this study variable grid size was used both in normal and tangential directions.

In order to test the general applicability of a correlation in the form of equation (1), it is convenient to rearrange (1) into the following two forms:

$$\left(\frac{Nu_{MC}}{Nu_N}\right)^n = 1 + \left(\frac{Nu_F}{Nu_N}\right)^n \quad (13)$$

$$\left(\frac{Nu_{MC}}{Nu_F}\right)^n = 1 + \left(\frac{Nu_N}{Nu_F}\right)^n \quad (14)$$

where

$$Nu_N = (0.6315 Pr^{1/5} Gr_x^{*1/5} / f_N(Pr)^{1/5} \quad \text{from [12]}$$

$$f_N(Pr) = [1 + (0.437/Pr)^{9/16}]^{16/9} \quad (15)$$

$$Nu_F = (0.464 Pr^{1/3} Re_x^{1/2}) / f_F(Pr) \quad \text{from [11]} \quad (16)$$

$$f_F(Pr) = [1 + (0.0205/Pr)^{2/3}]^{1/4}.$$

Figure 2 shows the numerical results for Nu_{MC} at various values of Prandtl number, plotted in terms of the groupings given by equations (13) and (14). The numerical results are for $Pr = 0.01, 0.1, 1, 10$ and 100.

Three representative values of n (2, 3 and 4) are considered. For $Nu_N/Nu_F < 1$, the r.m.s. differences between the numerical results and equation (14) are 1.7% and 2.3% for $n = 3$ and 4, respectively. For $Nu_N/Nu_F \geqslant 1$ the r.m.s. difference between the numerical results and equation (13) is 1.3% and 1.7% for $n = 3$ and 4, respectively. The following correlations are therefore recommended:

$$\frac{Nu_{MC}}{Nu_N} = \left(1 + \frac{Nu_F^3}{Nu_N^3}\right)^{1/3} \quad \text{if} \quad \frac{Nu_N}{Nu_F} \geqslant 1 \quad (17)$$

$$\frac{Nu_{MC}}{Nu_F} = \left(1 + \frac{Nu_N^3}{Nu_F^3}\right)^{1/3} \quad \text{if} \quad \frac{Nu_N}{Nu_F} < 1. \quad (18)$$

The same correlations are recommended in ref. [13]. However, the recommendation in ref. [13] was based primarily on the numerical results for $Pr = 1$. The

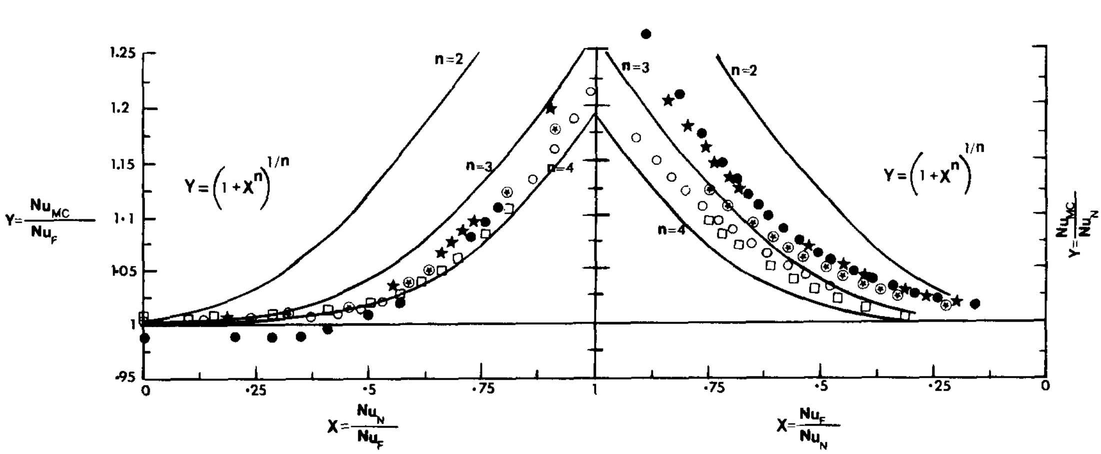

FIG. 2. A general correlation based on numerically obtained results for local heat transfer rates in laminar, mixed convection from a vertical, uniform flux surface. □ $Pr = 100$; ○ $Pr = 10$; ⊛ $Pr = 1$; ★ $Pr = 0.1$; ● $Pr = 0.01$.

numerical result for $Pr \neq 1$ in ref. [13] shows considerable scatter. Equation (17) indicates that for forced convection to have an effect of 5% or less on the Nusselt number

$$\varepsilon_m < 12.10 g^2(Pr) \tag{19}$$

where

$$g(Pr) \equiv \frac{f_F(Pr)}{[f_N(Pr)\, Pr^{2/3}]^{1/5}}. \tag{20}$$

Similarly, for the effect of natural convection to be less than 5% on the heat transfer, equation (18) gives

$$\varepsilon_m > 1.03 g^2(Pr). \tag{21}$$

Thus the effective mixed convection region is

$$1.03 < \frac{\varepsilon_m}{g^2(Pr)} < 12.10. \tag{22}$$

3.3. *Transition and turbulent regime—experimental results*

So far only the laminar flow data have been considered. However, laminar flows are generally susceptible to breakdown due to growth of ever-present disturbances. It is now well established [19] that such a breakdown is not abrupt. Rather, downstream of the first appearance of concentrated turbulence, the flow undergoes a gradual transition to full turbulence. A long transition region arises. Changes in the transport mechanisms occur and the heat transfer coefficient will deviate from its value corresponding to laminar flow. It is important to know the bounds of the transition region. The criteria for determining the beginning and end of the transition region are discussed in ref. [19].

The experiments considered here involved small, forced-convection effects for flow in the transition regime. The analysis of these data, based on considerations discussed in ref. [19], indicates that fully turbulent flow did not occur in these experiments. The only heat transfer data for a fully turbulent, vertical, mixed convection flow are those of Hall and Price [9]. Since there is no heat transfer correlation of turbulent, mixed convection, the data in ref. [9] were used here to develop such a correlation.

In ref. [9], downstream surface temperature distributions were measured for various levels of air velocity and input heat flux, at atmospheric pressure. These distributions are given graphically in ref. [9], making it possible to determine the values of corresponding local Nusselt numbers. The results are shown in Fig. 3. Also shown are data obtained from our experiments; these correspond to the transition region upstream.

The heat transfer correlation is based on the kind of general formulation used above in the laminar regime. It consists of combining the turbulent, forced and natural convection correlations. For turbulent, forced convection from a uniform flux surface, the correlation [20], for $Pr = 0.7$, is

$$Nu_{F,t} = 0.024 Re_x^{0.8}. \tag{23}$$

For turbulent, natural convection, a common form of

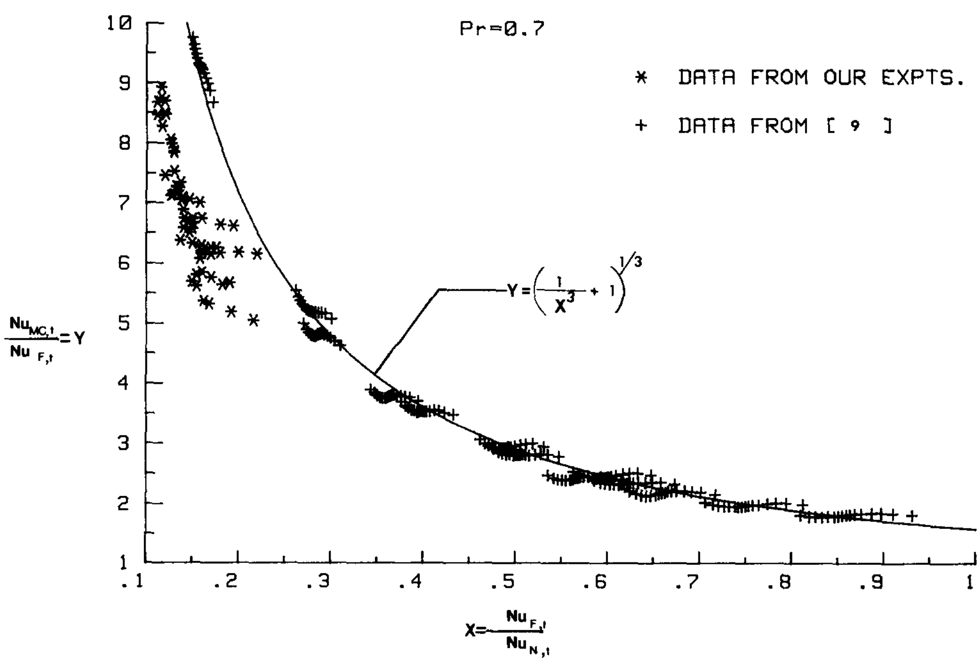

FIG 3. A correlation for heat transfer rates in turbulent, mixed convection from a vertical, uniform flux surface.

correlation is

$$Nu_{N,t} = CGr_x^{*1/4}. \tag{24}$$

However, as indicated in ref. [21], there is no consensus on the appropriate value of C. The three different values, 0.17, 0.21 and 0.16, are suggested by earlier investigators. It is recommended in ref. [21] that $C = 0.17$ be used since it yielded the best fit to that data.

The form of the turbulent mixed convection correlation was taken as

$$\frac{Nu_{MC,t}}{Nu_{F,t}} = \left[\left(\frac{Nu_{N,t}}{Nu_{F,t}}\right)^n + 1\right]^{1/n} = \left[\left(\frac{CGr_x^{*1/4}}{0.024Re_x^{0.8}}\right)^n + 1\right]^{1/4}. \tag{25}$$

A non-linear regression fit for the data from ref. [9], using $Nu_{F,t}$ as given in (23), yielded $C = 0.229$ and $n = 3$. The resulting correlation for turbulent, mixed convection is

$$\frac{Nu_{MC,t}}{Nu_{F,t}} = \left[\left(\frac{Nu_{N,t}}{Nu_{F,t}}\right)^3 + 1\right]^{1/3} = \left[\left(\frac{9.54Gr_x^{*1/4}}{Re_x^{0.8}}\right)^3 + 1\right]^{1/3}. \tag{26}$$

This relation is re-written as

$$\frac{Nu_{MC,t}}{Nu_{F,t}} = f\left(\frac{Nu_{F,t}}{Nu_{N,t}}\right) = f\left(\frac{Re_x^{0.8}}{Gr_x^{*1/4}}\right)$$

$$= F_1\left(\frac{Re_x}{Gr_x^{*0.3125}}\right) = F_1(\varepsilon_{m,t}). \tag{27}$$

As $Gr_x^* \to 0$, $\varepsilon_{m,t} \to \infty$ and equation (26) indicates that $Nu_{MC,t} \to Nu_{F,t}$. Similarly as $Re_x \to 0$, $\varepsilon_{m,t} \to 0$ and equation (26) indicates that $Nu_{MC,t} \to Nu_{N,t}$. Thus, $\varepsilon_{m,t}$ appears to be a legitimate mixed convection parameter, in that both limits are correctly obtained.

Recall that, in laminar regime, the mixed convection parameter was

$$\varepsilon_m \equiv \frac{Re_x}{(Gr_x^*/5)^{2/5}} = \frac{U_\infty}{U_c}.$$

That is, ε_m is equal to U_∞/U_c where U_c is a characteristic, laminar, natural convection velocity. Therefore, it is instructive to examine the parameter in (27), $\varepsilon_{m,t}$, in terms of U_∞ and $U_{c,t}$, where $U_{c,t}$ is a characteristic turbulent natural convection velocity. That is, may $\varepsilon_{m,t}$ be interpreted as

$$\varepsilon_{m,t} = \frac{Re_x}{Gr_x^{*0.3125}} = \frac{U_\infty x/\nu}{U_{c,t}x/\nu} = \frac{U_\infty}{U_{c,t}}. \tag{28}$$

To this end, some of the studies in turbulent, natural convection are next considered.

Based on scaling arguments, George and Kapp [22] have proposed a theory for developed, turbulent, natural convection. It is proposed that the boundary layer be treated in two parts. There is an outer region in which viscous and conduction terms are negligible. In the inner region the mean convection terms are negligible. The inner region is shown to be characterized by a constant heat flux. It is further shown that the inner region consists of conductive, viscous and buoyant sublayers. One kind of prediction of the model, is the form of the velocity and temperature profiles in the sublayers. In addition the heat transfer result from the model is in agreement with the experimental data reported in literature. Details are given in ref. [22].

Since the heat flux across the inner layer is constant and as the mean temperature level decreases, it may be surmised that temperature fluctuations are growing in this region at the expense of the mean flow. However, the action of convection in reducing the temperature level is felt only in the outer layer. It is therefore expected that the characteristic velocity, relevant to the heat transfer process, is the velocity corresponding to the outer layer. Thus $U_{c,t}$ should be at least proportional to the outer velocity scale, as defined in ref. [22]. From ref. [22] the outer velocity scale is U_0, where

$$U_0 \equiv \left(\frac{q''g\beta\delta_u}{\rho c_p}\right)^{1/3} \tag{29}$$

and

$$\delta_u \equiv \int_0^\infty \frac{\bar{U}}{\bar{U}_{max}}\,dy.$$

Equation (29) may be rewritten as

$$\frac{U_0 x}{\nu} = \frac{1}{Pr^{1/3}}\left(\frac{g\beta q'' x^3\delta_u}{k\nu^2}\right)^{1/3}.$$

The data from ref. [9] were for air, $Pr = 0.7$. Since $(0.7)^{1/3} \sim O(1)$, we may write

$$\frac{U_0 x}{\nu} = Gr_x^{*0.33}\left(\frac{\delta_u}{x}\right)^{1/3}. \tag{30}$$

Comparing equations (28) and (30), it is seen that if

$$\left(\frac{\delta_u}{x}\right)^{1/3} \simeq O(1) \tag{31}$$

then

$$\frac{U_{0,t}x}{\nu} = Gr_x^{*0.33} \simeq Gr_x^{*0.3125} = \frac{U_{c,t}x}{\nu}.$$

However, the data available in the literature for turbulent, natural convection in air is not extensive enough to test thoroughly the validity of (31). Nevertheless, based on the two data points available for δ_u [23], $(\delta_u/x)^{1/3} \simeq 0.3$. Thus it appears reasonable to surmise that the Nusselt number for turbulent mixed convection, suitably normalized, is a function solely of the ratio of the relevant characteristic velocity scales, $U_\infty/U_{c,t}$.

4. CONCLUSION

Experimental data are given for heat transfer in mixed convection adjacent to a vertical, uniform flux surface. The data span all the three flow regimes: laminar, transition and turbulence. Simple relations to correlate the heat transfer data are given. The method of combining the Nusselt number correlations of forced and natural convection, in formulating the Nusselt number correlation for mixed convection transport, is shown to be quite successful—even for fully turbulent flow. It has also been shown that the Nusselt number for mixed convection, normalized suitably, is solely a function of the ratio of relevant characteristic velocity scales, in both laminar and turbulent flow.

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TRANSFERT THERMIQUE PAR CONVECTION MIXTE SUR UN ECOULEMENT VERTICAL TRAVERSANT LA TRANSITION

Résumé—On considère le transfert thermique pour une convection mixte aidée sur des surfaces verticales à flux constant et pour des effets de convection forcée faible. Des relations simples sont proposées pour unifier les nouvelles données expérimentales. On obtient de nouveaux résultats pour un écoulement de transition entre les régimes laminaire et turbulent. Des expériences sont faites dans l'air à des pressions allant de 4,4 à 8 bar environ. La formule basée sur l'écoulement laminaire pour $Pr = 0{,}7$ est étendue à d'autres nombres de Prandtl à partir de l'intégration numérique des équations de transport. On montre que pour la convection mixte laminaire ou turbulente, le nombre de Nusselt peut être correctement représenté en employant des combinaisons convenables des formules correspondantes de transfert thermique pour les convections naturelle ou forcée. Le paramètre qui caractérise l'effet de convection mixte est trouvé différent en écoulement soit laminaire, soit turbulent. Néanmoins dans chaque région, le paramètre significatif est proportionnel au rapport des échelles de vitesse caractéristiques de la convection naturelle ou forcée.

WÄRMEÜBERGANG IN EINER VERTIKALEN STRÖMUNG BEIM ÜBERGANG VOM LAMINAREN ZUM TURBULENTEN GEBIET

Zusammenfassung—Es wurde der Wärmeübergang für die aufwärtsgerichtete Mischkonvektion an vertikalen Oberflächen mit konstanter Wärmestromdichte und für geringen Einfluß der erzwungenen Konvektion betrachtet. Eine einfache Beziehung wird vorgeschlagen, um die neuen experimentellen Daten zu korrelieren. Die neuen Daten für den Wärmeübergang wurden für eine Strömung ermittelt, die vom laminaren in den turbulenten Bereich übergeht. Die Experimente wurden mit Luft bei Drücken zwischen 4,4 und 8 bar durchgeführt. Die auf experimentellen Daten für laminare Strömung und für $Pr = 0{,}7$ basierende Korrelation wurde durch eine numerische Integration der Transportgleichungen auf andere Prandtl-Zahlen erweitert. Es wird gezeigt, daß für laminare und turbulente Mischkonvektion die Nusselt-Zahl erfolgreich korreliert werden kann, wenn man geeignete Kombinationen von entsprechenden Wärmeübergangsbeziehungen für erzwungene und natürliche Konvektion verwendet. Es ergab sich, daß der die Mischkonvektion charakterisierende Parameter bei laminarer und turbulenter Strömung verschieden ist. Jedoch ist in jedem dieser Bereiche der wesentliche Parameter proportional zum Verhältnis des geeigneten charakteristischen Geschwindigkeitsmaßstabes für erzwungene und natürliche Konvektion.

ТЕПЛОПЕРЕНОС СМЕШАННОЙ КОНВЕКЦИЕЙ ПРИ ВЕРТИКАЛЬНОМ ПЕРЕХОДНОМ ТЕЧЕНИИ

Аннотация—В настоящей статье рассматривается теплоперенос смешанной конвекцией от поверхностей вертикального однородного потока при незначительной роли вынужденной конвекции. Предложены простые соотношения для описания новых экспериментальных данных для теплопереноса при переходе от ламинарного течения к турбулентному. Выполнены эксперименты для воздуха в диапазоне (4,4–8) бар. Полученная на основе экспериментальных данных для ламинарного потока при $Pr = 0{,}7$ зависимость с помощью численного интегрирования уравнений переноса обобщена на другие значения числа Прандтля. Показано, что и при ламинарной, и при турбулентной смешанной конвекции критериальную зависимость для числа Нуссельта можно успешно описать, используя соответствующие критериальные зависимости для теплопереноса при вынужденной и естественной конвекции. Установлено, что параметр, характеризующий влияние смешанной конвекции, различен для ламинарного и турбулентного течения. Однако в обоих режимах указанный параметр пропорционален отношению характерных масштабов скоростей, используемых при описании вынужденной и естественной конвекции.